

The story of the n-fold integrals

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This story shows that the solution of some mathematical problems can determine one's mathematical life although these problems have nothing to do with one's later mathematical research.

In winter semester 1959/60 I started to study mathematics at the Universität Wien. One year later I attended the „Hochschulkurs für moderne Rechentechnik“ at the Technische Hochschule Wien (nowadays that would be a bachelor's study of informatics).

In my fifth semester, in winter semester 1961/62, I visited a seminar of Prof. Nikolaus Hofreiter on Cassels' book "An Introduction to the Geometry of Numbers" [1]. In this seminar each student had to lecture one hour on the contents of about one page of this book. Before lecturing the student had to submit a written report of his talk to the professor. On my page appeared the harmless looking line, which turned out to be an n-fold integral:

$$\int_{\mathfrak{P}} \left\{ \sum_{u \equiv \Lambda} \psi(\lambda^{-1}u + \lambda^{-1}t) \right\} dt,$$

where \mathfrak{P} is a fundamental parallelepiped of Λ .

In my studies only double integrals were dealt with. After one week I finished around ten pages of my written report including all necessary proofs by induction. To lecture on these ten pages would have needed much more than one hour and therefore Prof. Hofreiter told me to skip the proof of the integral. But he acknowledged my proof and I got the best grade.

It is remarkable that this seminar was attended by four students who became lifelong friends and later-on full professors: Peter Gruber (Universität Linz, Technische Universität Wien), Wilfried Imrich (Montanuniversität Leoben), Werner Kuich (Technische Universität Wien), Hermann Maurer (University of Calgary, Universität Karlsruhe, Technische Universität Graz).

At the beginning of my eighth semester, it was the summer semester 1963, I asked Prof. Edmund Hlawka to give me a topic for a dissertation. After having checked my certificates he was willing to give me one in the theory of uniform distributions, but told me to wait until he would have received the reprint of his latest paper [2]. Noticing my studies at the Technische Hochschule Wien, he mentioned that the IBM Forschungsgruppe Wien had a vacancy for a mathematician. I visited Prof. Zemanek, the head of IBM Forschungsgruppe Wien, who informed me that Dr. Kurt Walk, a department manager, needed a solution to a mathematical problem for a paper to be published [6]. Dr. Walk mentioned something about entropy of languages and inductive logics, which I did not understand. But then he reduced the problem to the solution of the n-fold integral:

$$- \int \cdots \int_{B_1} \frac{1}{s^{\mu-1}} \frac{(\mu k - 1)!}{((k - 1)!)^\mu} \prod_{0 \leq i \leq \mu-1} \frac{s_i^{k-1}}{s} \sum_{0 \leq j \leq \mu-1} \frac{s_j}{s} \text{ld} \frac{s_j}{s} \prod_{0 \leq \ell \leq \mu-2} ds_\ell,$$

where $\sum_{0 \leq i \leq \mu-1} s_i = s$ and B_1 is a parallelepiped.

Before I left he gave me the integral tables of Ryshik and Gradstein [5]. At this time I did not know anything about integral tables, but I took the book and started to work. After three days I found the solution [4] and was employed as a researcher at the IBM Forschungsgruppe Wien (later on called IBM Laboratory Vienna). This was the only institution in Austria which made research in language and automata theory. The solution of this n-fold integral, which was a kind of an entrance exam, determined the research direction of my whole life although n-fold integrals have nothing in common with language or automata theory.

At the end of the summer semester Prof. Hlawka gave me the reprint of his paper and asked me to read it. After some weeks I visited again Prof. Hlawka. He talked about five minutes on the

topic of my dissertation—and these five minutes were enough to occupy me for three months with mathematical research. That happened again twice and it was admirable how quickly Prof. Hlawka recognized my problems and found answers to them. Then it turned out that half of my dissertation [3] dealt with the solution of four n -fold integrals.

I had to solve n -fold integrals three times in my life. Each time the solution of these integrals influenced my mathematical life considerably. After my dissertation I never again had to do with n -fold integrals.

References

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